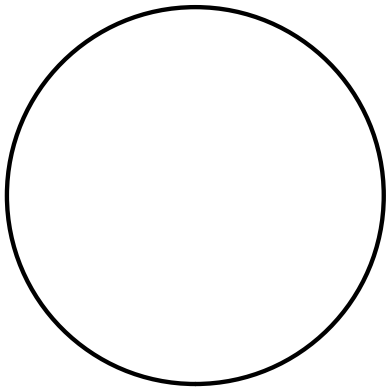
6.2: Addition and Subtraction of Rational Numbers

Definition: If $\frac{a}{b}$ and $\frac{c}{b}$ are rational numbers, then $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$.

Example: Draw a figure and a number line to represent $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$.



Example: Find a method of evaluating $\frac{1}{3} + \frac{1}{4}$.

6.2 Notes

Theorem: If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.

(Note: In practice, we usually find the least common denominator instead of just using this formula.)

Proof:

Example: Calculate the following sums. Simplify your answers.

(a) $\frac{3}{10} + \frac{4}{15}$

(b) $\frac{3}{9} + \frac{2}{6}$

6.2 Notes

Definition: A mixed number is a number of the form $a\frac{b}{c}$, where a is an integer and $\frac{b}{c}$ is a proper fraction. The notation means $a\frac{b}{c} = a + \frac{b}{c}$.

A mixed number is a rational number, so we should be able to write it as $\frac{a}{b}$.

Example: Write the following numbers in the $\frac{a}{b}$ form.

(a) $2\frac{1}{4}$

(b) $-3\frac{2}{5}$

Example: Change the following fractions to mixed numbers.

(a) $\frac{22}{7}$

(b) $\frac{64}{19}$

6.2 Notes

Example: Calculate the following sums. Leave your answers as mixed numbers.

(a) $2\frac{3}{7} + 1\frac{11}{14}$

(b) $2\frac{1}{2} + 3\frac{2}{3}$

Which of our number properties are represented in the rational numbers over addition?

Closure:

Commutative:

Associative:

Identity:

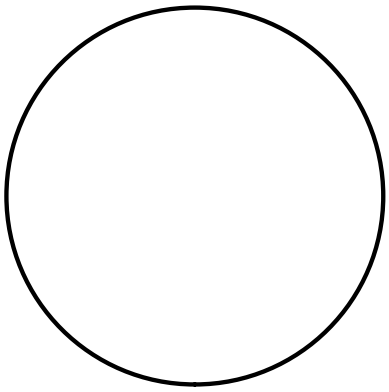
Inverse:

6.2 Notes

Definition: If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then $\frac{a}{b} - \frac{c}{d}$ is the unique rational number $\frac{e}{f}$ such that $\frac{a}{b} = \frac{c}{d} + \frac{e}{f}$.

Note: An easy formula that we can get from this is $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$.

Example: Draw a figure to represent $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$.



Theorem: If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$.

(Note: In practice, we usually find the least common denominator instead of just using this formula.)

Proof:

Example: Calculate the following. Simplify your answer.

(a) $\frac{3}{5} - \frac{1}{4}$

(b) $\frac{13}{16} - \frac{7}{30}$

Example: Calculate the following. Simplify your answer.

(a) $2\frac{1}{6} - 1\frac{9}{20}$

(b) $3\frac{1}{2} - 1\frac{5}{8}$